A Framework for Efficient Implementation and Effective Visualization of Dempster-Shafer Belief Theoretic Computations for Reasoning Under Uncertainty

Ph.D. Dissertation Proposal Defense

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Outline

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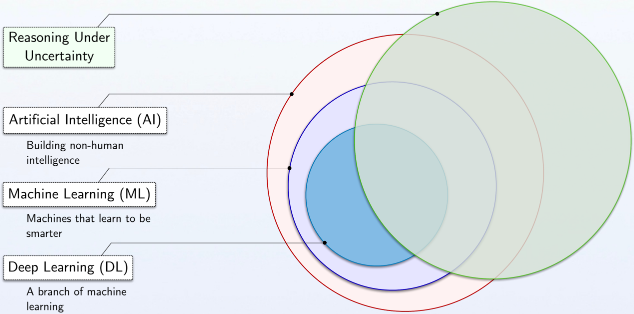
Expert Systems

- Beat champion players at Chess and Go[1]
- Allow AI assistants to book appointments over the phone[2]
- Diagnose and predict disease in health care[3]
- Drive autonomous vehicles[4]

Reasoning Under Uncertainty

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." - Albert Einstein

- Expert systems are still prone to collapse due to the difficulty in replicating complex environments[5].



Dempster-Shafer (DS) Belief Theory

- To accommodate uncertainty and data imperfections intelligently, we need to have effective models to capture them.

- The Dempster-Shafer (DS) belief theory is a framework for handling a wide variety of data imperfections[6].

  - First introduced by Dempster[7] in the context of statistical inference.

  - Then developed by Shafer[8] into a general framework for uncertainty modeling.

- A foundation for many important developments, including the transferable belief model (TBM)[9] and the theory of hints[10].

Motivation

- DS theory offers **greater expressiveness and flexibility** for modeling a wide variety of data imperfections.
- But the **main criticism** is that DS theoretic (DST) operations involve a **higher computational complexity**[6].
  - Computing the DST conditionals, DST belief functions, are non-deterministic polynomial-time hardness (NP-hard) problems[11].
- This is further exacerbated by the **absence of a flexible and scalable platform for visualizing** complex operations in DS theory.
- **Developing an efficient computational framework** is of critical importance if we are to harness the strengths of DS theory and make it more widely applicable in practice.

Challenges

1. Making Exact Computation of DST Quantities Feasible
   - Several **approximation methods** are available[12]:
     - But they compromise the quality of the results to gain computational efficiency.
     - Some lack the ability to be extended for DST conditional computations.
   - **Exact computation of conditionals is of paramount importance:** Quality of results generated from DST strategies depend directly on the precision of the conditional.
2. Developing a Feasible and Scalable Computational Framework
   - There is no widely accepted computationally feasible generalized framework to represent DST models and carry out DST operations.
   - A thoughtful discussion about data structures and algorithms for efficient DST computations is still lacking.

Challenges

3. Handling Large Frames of Discernment (FoDs)
   - DST implementations in current use are limited to computations on smaller FoDs.
4. Efficient Computation of DST Conditionals: There are two notions of DST conditionals to be dealt with.
   - **Dempster's conditional:** Perhaps the most extensively utilized DST conditional notion[8].
   - **Fagin-Halpern (FH) conditional:** The most natural generalization of the probabilistic conditional notion[13].

Challenges

4. Efficient Computation of DST Conditionals ...

- Dempster's conditional computation: **Specialization matrix approach**[14].
  - Cannot be used to compute the FH conditional.
  - Employs a $2^{|\Theta|} \times 2^{|\Theta|}$-sized stochastic matrix and a $2^{|\Theta|} \times 1$-sized vector containing the focal elements ($|\Theta|$ = cardinality of the FoD).
  - Computational complexity and space complexity are both $\mathcal{O}(2^{|\Theta|} \times 2^{|\Theta|})$.
  - Over 1800 CPU years for an FoD of size 30 and over 15 CPU hours for an FoD of size 20 (assuming 10 million computational iterations per second).
- FH conditional computation: No existing strategy.
  - **Conditional core theorem (CCT)**[15] can be used to identify (but not compute) propositions that retain non-zero support after FH conditioning.
  - But its computational complexity becomes $\mathcal{O}(2^{|\Theta|} \times 2^{|\Theta|})$ for large 'dense' FoDs.

Challenges

5. Visualization and Analysis of Complex DST Operations: No existing effective mechanism.
   - The ability to visualize complex DST computations and simulations is invaluable
     - to ensure the integrity of representation and reasoning,
     - to provoke insights that can lead to improvements in computational performance.

Contributions

1. Scalable Generalized Computational Framework
2. Implicit Index Calculation Mechanism
3. Efficient Computation of DST Operations
4. Efficient Computation of DST Conditionals
5. Computational Libraries
6. Effective Visualization Tools

Outline

DST Basic Notions

Symbol	Meaning
$\Theta = \{\theta_0, \ldots, \theta_{n-1}\}$	*Frame of discernment (FoD),* the set of all possible mutually exclusive and exhaustive propositions. Note $n = \lvert\Theta\rvert$.
θ_i	*Singletons,* i.e., the lowest level of discernible information.
$\overline{A}$	*Complement* of $A \subseteq \Theta$, i.e., those singletons that are not in A.
$m(\cdot)$	*Basic belief assignment (BBA)* or *mass assignment* $m : 2^\Theta \mapsto [0, 1]$ where $\sum_{A \subseteq \Theta} m(A) = 1$ and $m(\emptyset) = 0$.
Focal element	A proposition that receives a non-zero mass.
$\mathfrak{F}$	*Core,* the set of focal elements.
$\mathcal{E} = \{\Theta, \mathfrak{F}, m\}$	*Body of evidence (BoE).*

Belief, Plausibility and Commonality

- From now on, we assume that the BoE is $\mathcal{E} = \{\Theta, \mathfrak{F}, m(\cdot)\}$.

Definition 1 (Belief)

Belief assigned to $A \subseteq \Theta$ is $Bl : 2^\Theta \mapsto [0,1]$ where $Bl(A) = \sum_{B \subseteq A} m(B)$. ■

Definition 2 (Plausibility)

Plausibility assigned to $A \subseteq \Theta$ is $Pl : 2^\Theta \mapsto [0,1]$ where $Pl(A) = 1 - Bl(\overline{A})$. ■

Definition 3 (Commonality)

Commonality function of $A \subseteq \Theta$ is $Q : 2^\Theta \mapsto [0,1]$ where $Q(A) = \sum_{A \subseteq B \subseteq \Theta} m(B)$. ■

DST Conditionals

Definition 4 (Dempster's conditional)

Conditional belief $Bl(B\|A) : 2^\Theta \mapsto [0,1]$ of B given A is

$$Bl(B\|A) = \frac{Bl(\overline{A} \cup B) - Bl(\overline{A})}{1 - Bl(\overline{A})}, \text{ whenever } Bl(\overline{A}) \neq 1, \text{ or equivalently, } Pl(A) \neq 0.$$

The *conditional mass* $m(B\|A) : 2^\Theta \mapsto [0,1]$ of B given A is

$$m(B\|A) = \begin{cases} \dfrac{\sum_{C \subseteq \overline{A}} m(B \cup C)}{1 - Bl(\overline{A})}, & \text{for } \emptyset \neq B \subseteq A; \\ 0, & \text{otherwise.} \end{cases}$$

■

Definition 5 (Fagin-Halpern (FH) conditional)

Conditional belief $Bl(B|A)$ of B given A is

$$Bl(B|A) = \frac{Bl(A \cap B)}{Bl(A \cap B) + Pl(A \cap \overline{B})}, \text{ whenever } Bl(A) > 0.$$

■

Outline

REGAP: REcursive Generation of and Access to Propositions

Start with $\{\varnothing\}$ element

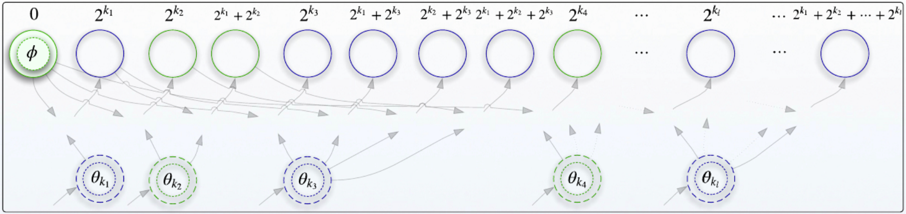


Figure: REGAP: Start with $\{\varnothing\}$.

- Consider the FoD $\Theta = \{\theta_0, \theta_1, \ldots, \theta_{n-1}\}$.
- Suppose we desire to determine the belief potential associated with $A = \{\theta_{k_1}, \theta_{k_2}, \ldots, \theta_{k_\ell}\} \subseteq \Theta$.
- The REGAP property allows us to recursively generate the propositions that are relevant for this computation: Start with $\{\varnothing\}$.

REGAP

Insert singleton $\{\theta_{k_1}\}$

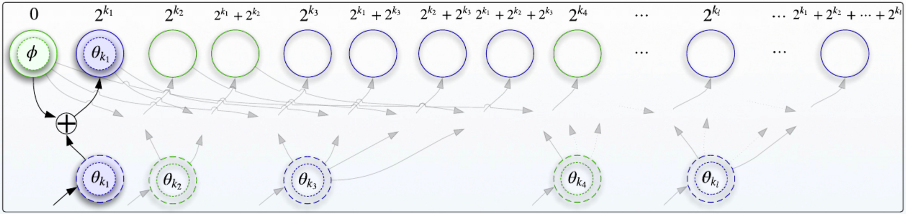


Figure: REGAP: Insert $\{\theta_{k_1}\}$.

- First insert the singleton $\{\theta_{k_1}\} \in A$. Only one proposition is associated with this singleton, viz.,

$$\{\varnothing\} \cup \{\theta_{k_1}\} = \{\theta_{k_1}\}.$$

REGAP

Inser singleton $\{\theta_{k_2}\}$

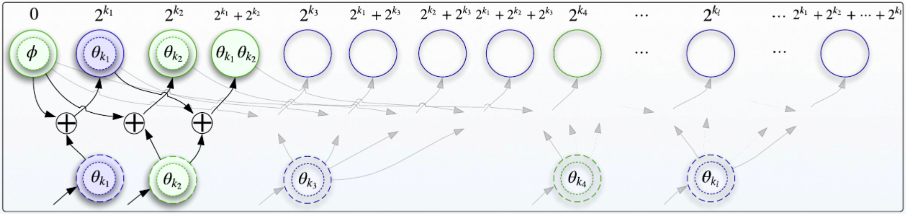


Figure: REGAP: Insert $\{\theta_{k_2}\}$.

- Next insert another singleton $\{\theta_{k_2}\} \in A$. The new propositions that are associated with this singleton are

$$\{\varnothing\} \cup \{\theta_{k_2}\} = \{\theta_{k_2}\}, \ \ \{\theta_{k_1}\} \cup \{\theta_{k_2}\} = \{\theta_{k_1}, \theta_{k_2}\}.$$

REGAP

Insert singleton $\{\theta_{k_3}\}$

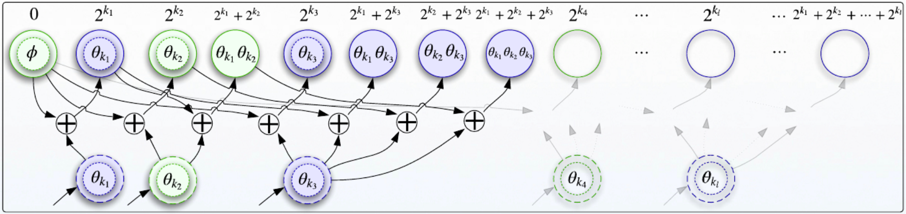


Figure: REGAP: Insert $\{\theta_{k_3}\}$.

- Inserting another singleton $\{\theta_{k_3}\} \in A$ brings the new propositions

$$\{\varnothing\} \cup \{\theta_{k_3}\} = \{\theta_{k_3}\}, \qquad \{\theta_{k_1}\} \cup \{\theta_{k_3}\} = \{\theta_{k_1}, \theta_{k_3}\},$$
$$\{\theta_{k_2}\} \cup \{\theta_{k_3}\} = \{\theta_{k_2}, \theta_{k_3}\}, \quad \{\theta_{k_1}, \theta_{k_2}\} \cup \{\theta_{k_3}\} = \{\theta_{k_1}, \theta_{k_2}, \theta_{k_3}\}.$$

- In essence, the new propositions associated with a new singleton can be recursively generated by adding the new singleton to each existing proposition.

REGAP

Generalized representation

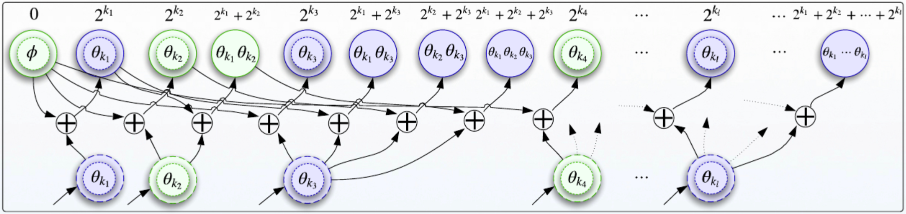


Figure: REGAP = REcursive Generation of and Access to Propositions.

- When $A = \Theta$, REGAP generates the powerset of the FoD Θ.
- These recursively generated propositions can be formulated as a vector, a matrix, or a tree, and utilized to represent a dynamic BoE.
- From now on, we use the following notation:

$$REGAP(A) = \text{all the propositions that are required to compute } Bl(A).$$

DS-Vector: Vector Representation of a Dynamic BoE

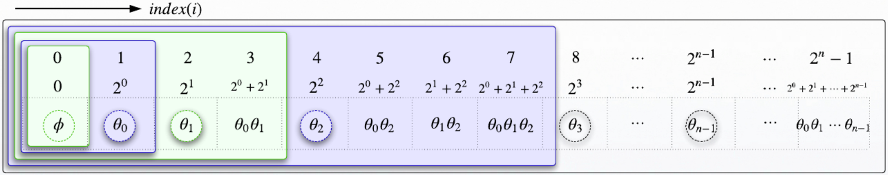


Figure: DS-Vector: Vector representation of a dynamic BoE.

- Rectangles represent the recursive steps of dynamic BoE generation.
- Propositions are represented by **implicit contiguous indexes**. So, no memory allocation is needed to store a proposition.
- Memory allocation is needed only to store the required belief potentials (or, more generally, mass, belief, plausibility, or commonality values).

DS-Matrix: Matrix Representation of a Dynamic BoE

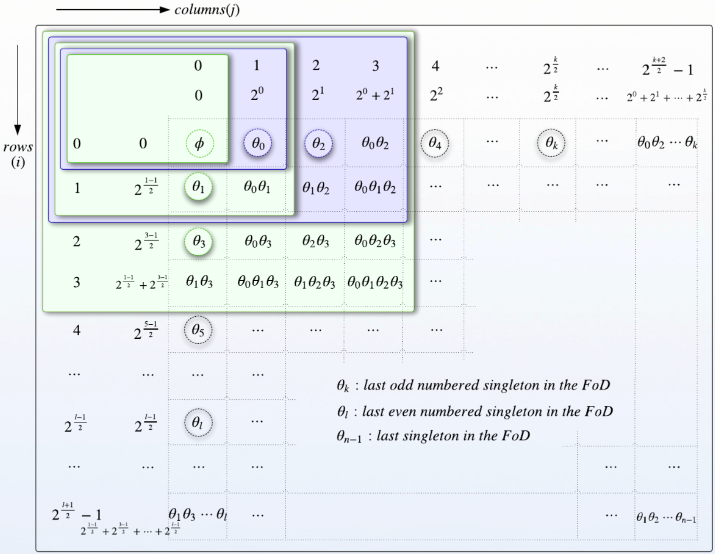


Figure: DS-Matrix: Matrix representation of a dynamic BoE.

DS-Tree: Perfectly Balanced Binary Tree Representation of a Dynamic BoE

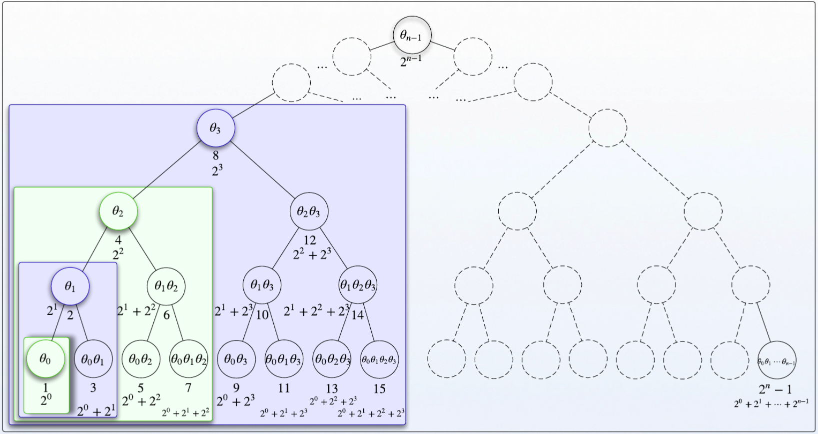


Figure: DS-Tree: Perfectly balanced binary tree representation of a dynamic BoE.

Arbitrary Belief Computations

DS-Matrix Version

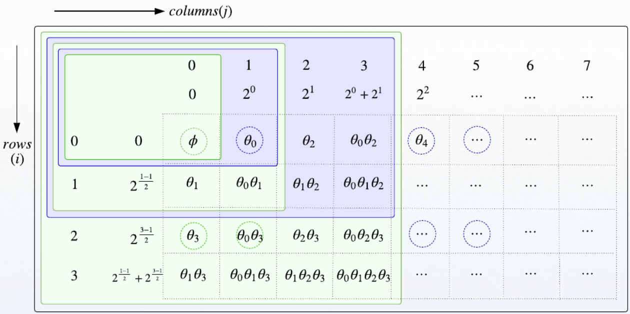


Figure: **Belief Calculation:** Propositions related to $Bl(A)$ computation when $A = \{\theta_0, \theta_3, \theta_4\}$, and $\Theta = \{\theta_0, \theta_1, \theta_2, \theta_3, \theta_4\}$.

- $REGAP(A)$ generates the propositions relevant to the computation of $Bl(A)$.
  - Belief computation is performed by accessing only the subset propositions.
  - Time complexity: $\mathcal{O}(2^{|A|})$.

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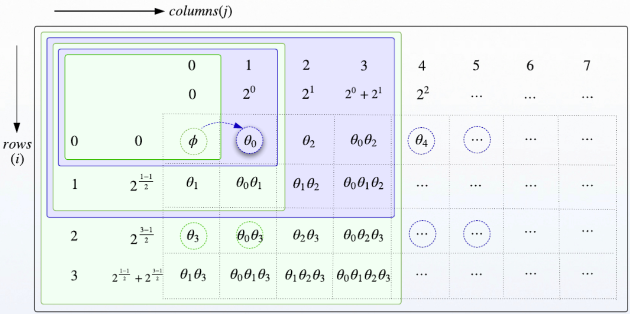

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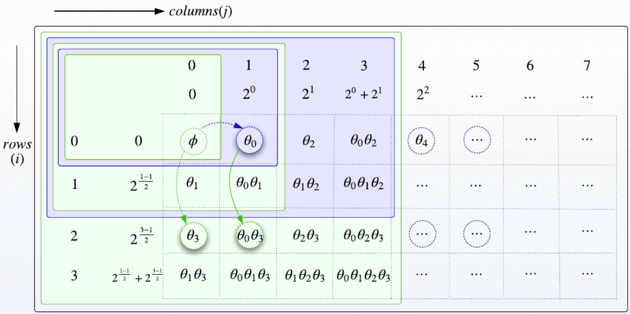

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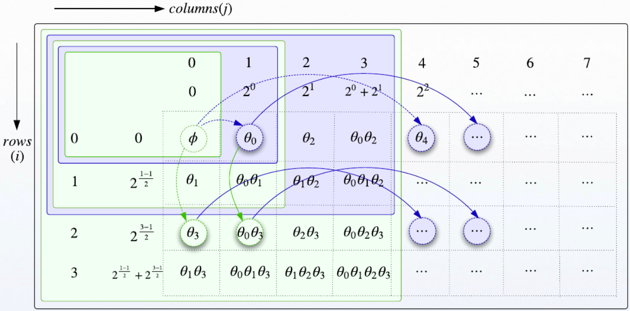

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- $REGAP(A)$ generates those propositions relevant to the computation of $Bl(A)$.
  - Belief computation is performed by accessing only the subset propositions.
  - Time complexity: $\mathcal{O}(2^{|A|})$.

Arbitrary Plausibility and Commonality Computations

DS-Matrix Version

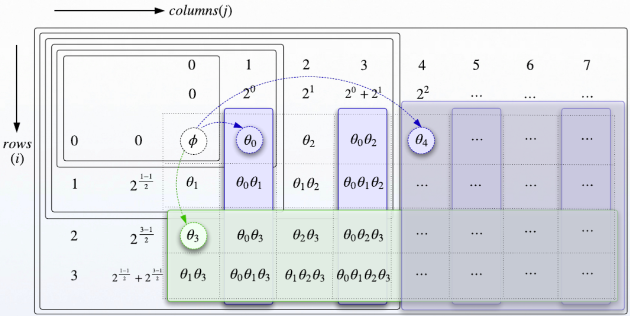


Figure: **Plausibility Calculation:** Propositions related to $Pl(A)$ computation when $A = \{\theta_0, \theta_3, \theta_4\}$, and $\Theta = \{\theta_0, \theta_1, \theta_2, \theta_3, \theta_4\}$.

- Computing the plausibility $Pl(A)$: Use $REGAP(\overline{A})$ to compute $Bl(\overline{A})$ and use the equation $Pl(A) = 1 - Bl(\overline{A})$.
- Computing the commonality $Q(A)$: Append the proposition A to all propositions generated from $REGAP(\overline{A})$ and apply the belief computation algorithm.

Experiments

Average CPU time of accessing a proposition (μs)

FoD Size	Max. $|\mathfrak{F}|$	DS-Vector	DS-Matrix	List Struct.
2	3	0.379	0.393	0.465
4	15	0.400	0.412	0.510
6	63	0.410	0.454	0.739
8	255	0.443	0.449	1.541
10	1023	0.433	0.496	4.632
12	4095	0.465	0.493	16.906
14	16383	0.465	0.527	67.242
16	65535	0.495	0.517	268.443
18	262143	0.529	0.560	1124.0600
20	1048575	0.575	0.629	4609.3700

- Machine used: Macintosh desktop computer running Mac OS X 10.11.3, with 2.9GHz Intel Core i5 processor and 8GB of 1600MHz DDR3 RAM.
- For each FoD size, the core of focal elements was randomly chosen.
- DST operation was computed for 100,000 randomly chosen propositions from the FoD.

Experiments

Average CPU time of belief computation (μs)

FoD Size	Max. $|\mathfrak{F}|$	DS-Vector	DS-Matrix	List Struct.
2	3	0.373	0.362	0.450
4	15	0.378	0.376	0.531
6	63	0.415	0.450	0.833
8	255	0.453	0.508	1.779
10	1023	0.525	0.663	5.529
12	4095	0.655	0.923	20.757
14	16383	0.884	1.314	81.196
16	65535	1.340	2.159	325.930
18	262143	2.107	3.510	1373.110
20	1048575	3.963	6.210	5448.170

- A new computational library, which we refer to as *BCL (Belief Computation Library)*[16] is developed and utilized in the simulations[17].

Outline

Alternate Expressions for Conditionals

$$\mathcal{S}(A;B) = \sum_{\substack{\emptyset \neq C \subseteq A;\\ \emptyset \neq D \subseteq B}} m(C \cup D)$$

Sum of all masses of propositions that 'straddle' both $A \subseteq \Theta$ and $B \subseteq \Theta$.

$$\mathcal{T}(A;B) = \sum_{C \subseteq A} m(C \cup B)$$

Sum of all masses of propositions in $A \subseteq \Theta$ that strictly 'straddle' proposition $B \subseteq \Theta$.

Proposition 1

Take $A \subseteq \Theta$. For $B \subseteq \Theta$, consider the mappings $\Gamma_A : 2^\Theta \mapsto [0,1]$ and $\Pi_A : 2^\Theta \mapsto [0,1]$, where

$$\Gamma_A(B) = \sum_{\emptyset \neq X \subseteq \overline{A}} m((A \cap B) \cup X) \text{ and } \Pi_A(B) = \sum_{Y \subseteq (A \cap B)} \Gamma_A(Y).$$

Then the following are true:

- $\Gamma_A(A \cap B) = \Gamma_A(B)$ and $\Pi_A(A \cap B) = \Pi_A(B)$. So, w.l.o.g., take $B \subseteq A$.
- $\Gamma_A(\emptyset) = \Pi_A(\emptyset) = Bl(\overline{A})$.
- $\Gamma_A(B) = \mathcal{T}(\overline{A}; A \cap B) - m(A \cap B)$.
- $\Pi_A(B) = \Pi_A(\emptyset) + \mathcal{S}(\overline{A}; A \cap B)$.

■

Alternate Expressions for Conditionals

- We use the **DS-Conditional-One** computational model to compute the following:
  - Dempster's and FH conditional beliefs of an arbitrary proposition.
  - Dempster's conditional masses of an arbitrary proposition.
- To proceed, we employ the following alternate expressions:

Proposition 2 (Propositions for Dempster's Conditional Belief and Mass)

Take $A \subseteq \Theta$ s.t. $Bl(\overline{A}) \neq 1$. Then, $Bl(B \| A)$ and $m(B \| A)$ can be expressed as

$$Bl(B\|A) = \frac{Bl(A \cap B) + \mathcal{S}(\overline{A}; A \cap B)}{1 - Bl(\overline{A})}; \quad m(B\|A) = \begin{cases} \dfrac{\mathcal{T}(\overline{A}; A \cap B)}{1 - Bl(\overline{A})}, & \text{for } \emptyset \neq B \subseteq A; \\ 0, & \text{otherwise.} \end{cases}$$

■

Proposition 3 (Propositions for FH Conditional Belief)

Take $A \subseteq \Theta$ s.t. $Bl(A) > 0$. Then, $Bl(B|A)$ can be expressed as

$$Bl(B|A) = \frac{Bl(A \cap B)}{1 - Bl(\overline{A}) - \mathcal{S}(\overline{A}; A \cap B)}.$$

■

DS-Conditional-One Computational Model

- Note that we need only the following four quantities to compute the Dempster's conditional beliefs/masses and FH conditional beliefs:

$$\underbrace{Bl(\overline{A})}_{REGAP(\overline{A})} \; ; \; \underbrace{Bl(A \cap B)}_{REGAP(A \cap B)}; \quad \underbrace{\mathcal{S}(\overline{A}; A \cap B);}_{REGAP(\overline{A}) \otimes REGAP(A \cap B)} \quad \underbrace{\mathcal{T}(\overline{A}; A \cap B).}_{(REGAP(\overline{A}) \otimes (A \cap B)) + (A \cap B)}$$

- To explain how the DS-Conditional-One model allows us to easily identify these quantities, let

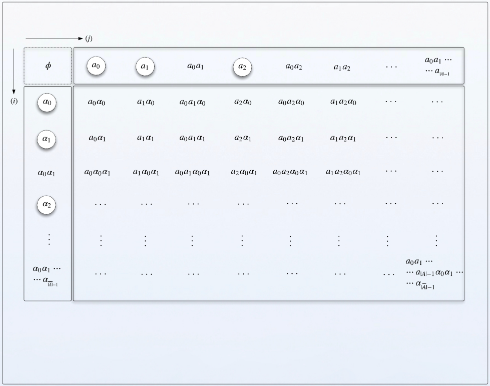
$$\begin{array}{ll} \text{FoD:} & \Theta = \{\theta_0, \theta_1, \ldots, \theta_{n-1}\}; \\ \text{conditioning proposition:} & A = \{a_0, a_1, \ldots, a_{|A|-1}\},\; a_i \in \Theta; \\ \text{its complement:} & \overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\},\; \alpha_i \in \Theta; \\ \text{conditioned proposition:} & B = \{a_0, a_2\}. \end{array}$$

DS-Conditional-One Computational Model

- Construct the DS-Matrix as follows:
  - **First row:** conditioning proposition $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$.
  - **First column:** its complement $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.

- We can now directly identify $REGAP(A \cap B)$, $REGAP(\overline{A})$, $REGAP(\overline{A}) \otimes REGAP(A \cap B)$, $(REGAP(\overline{A}) \otimes (A \cap B)) + (A \cap B)$.

- We can also directly identify $REGAP(A)$, $REGAP(\overline{A}) \otimes REGAP(A)$, $\Gamma_A(C), \forall C \subseteq B$.



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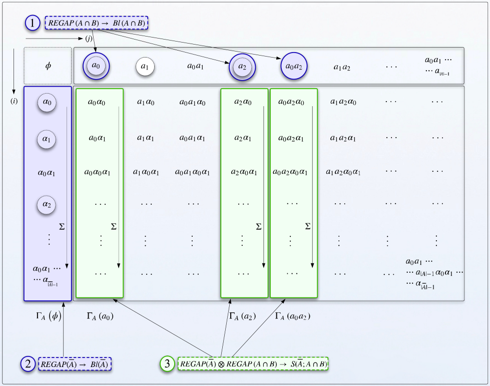

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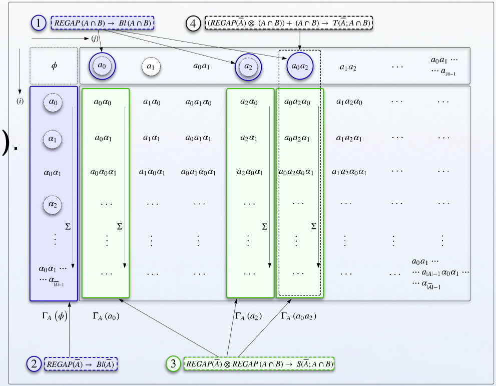

DS-Conditional-One Computational Model

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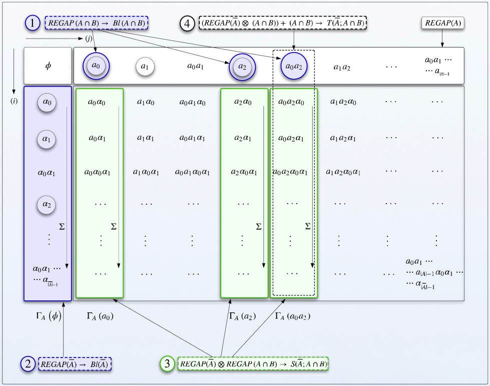
  - First row: conditioning proposition $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$.
  - First column: its complement $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.

- We can now directly identify $REGAP(A \cap B)$, $REGAP(\overline{A})$, $REGAP(\overline{A}) \otimes REGAP(A \cap B)$, $(REGAP(\overline{A}) \otimes (A \cap B)) + (A \cap B)$.

- We can also directly identify $\boldsymbol{REGAP(A)}$, $REGAP(\overline{A}) \otimes REGAP(A)$, $\Gamma_A(C), \forall C \subseteq B$.

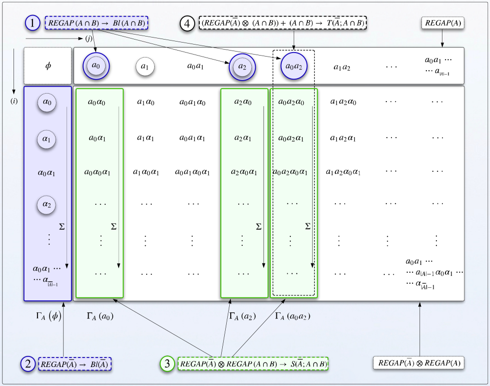


DS-Conditional-One Computational Model

- Construct the DS-Matrix as follows:
  - First row: conditioning proposition $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$.
  - First column: its complement $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.

- We can now directly identify $REGAP(A \cap B)$, $REGAP(\overline{A})$, $REGAP(\overline{A}) \otimes REGAP(A \cap B)$, $(REGAP(\overline{A}) \otimes (A \cap B)) + (A \cap B)$.

- We can also directly identify $REGAP(A)$, $\boldsymbol{REGAP(\overline{A}) \otimes REGAP(A)}$, $\Gamma_A(C), \forall C \subseteq B$.



DS-Conditional-One Computational Model

- Look at Propositions 2 and 3.

Use	To Compute	Complexity	
		Time	**Space**
Dempster's and FH Conditional Belief of an Arbitrary Proposition:			
(1) $REGAP(A \cap B)$	$Bl(A \cap B)$	$\mathcal{O}(2^{\lvert A \cap B \rvert})$	$\mathcal{O}(2^{\lvert \Theta \rvert})$
(2) $REGAP(\overline{A})$	$Bl(\overline{A}) = \Gamma_A(\varnothing)$	$\mathcal{O}(2^{\lvert \overline{A} \rvert})$	$\mathcal{O}(2^{\lvert \Theta \rvert})$
(3) $REGAP(\overline{A}) \otimes REGAP(A \cap B)$	$\mathcal{S}(\overline{A}; A \cap B)$	$\boldsymbol{\mathcal{O}(2^{\lvert \overline{A} \rvert + \lvert A \cap B \rvert})}$	$\mathcal{O}(2^{\lvert \Theta \rvert})$
Dempster's Conditional Mass of an Arbitrary Proposition:			
(2) $REGAP(\overline{A})$	$Bl(\overline{A}) = \Gamma_A(\varnothing)$	$\mathcal{O}(2^{\lvert \overline{A} \rvert})$	$\mathcal{O}(2^{\lvert \Theta \rvert})$
(4) $(REGAP(\overline{A}) \otimes (A \cap B)) + (A \cap B)$	$\mathcal{T}(\overline{A}; A \cap B)$	$\boldsymbol{\mathcal{O}(max(2^{\lvert \overline{A} \rvert}, \lvert A \cap B \rvert))}$	$\mathcal{O}(2^{\lvert \Theta \rvert})$

Outline

Alternate Expressions for Conditionals

- We use **DS-Conditional-All** computational model to compute the following:
  - Dempsters' and FH conditional beliefs of all propositions,
  - Dempsters' conditional masses of all propositions.

Proposition 4 (Propositions for Dempster's Conditional Belief and Mass)

Take $A \subseteq \Theta$ s.t. $Bl(\overline{A}) \neq 1$. Then, $Bl(B\|A)$ and $m(B\|A)$ can be expressed as

$$Bl(B\|A) = \frac{Bl(A \cap B) + \Pi_A(A \cap B) - \Gamma_A(\{\varnothing\})}{1 - \Gamma_A(\{\varnothing\})}; \quad m(B\|A) = \frac{m(A \cap B) + \Gamma_A(A \cap B)}{1 - \Gamma_A(\{\varnothing\})}. \qquad \blacksquare$$

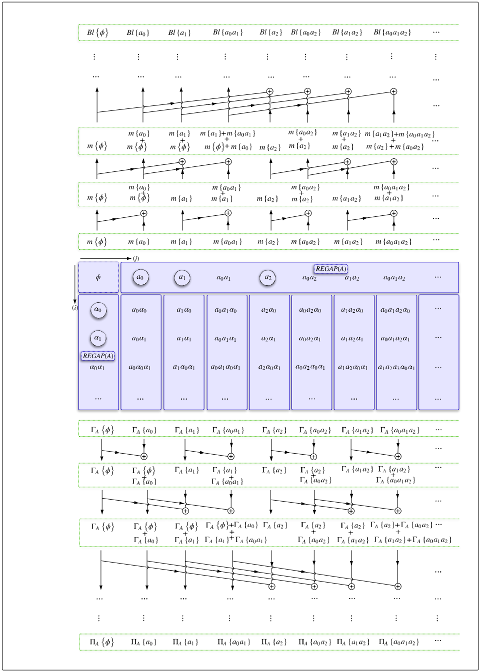
Proposition 5 (Propositions for FH Conditional Belief)

Take $A \subseteq \Theta$ s.t. $Bl(A) > 0$. Then, $Bl(B|A)$ can be expressed as

$$Bl(B|A) = \frac{Bl(A \cap B)}{1 - \Pi_A(A \cap B)}. \qquad \blacksquare$$

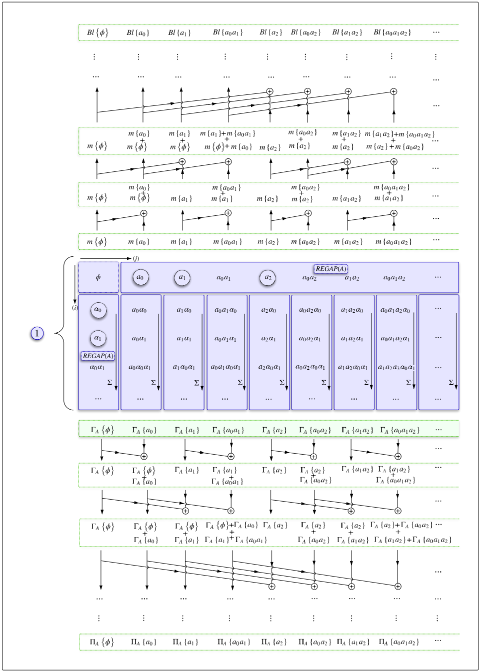
DS-Conditional-All Model

- Construct the DS-Matrix as before with $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$ and $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.
- Utilize the fast Möbius transformation (FMT)[18,19].
- Perform following computations: $\Gamma_A(B), \forall B \subseteq A$, $\Pi_A(B)$ values from $\Gamma_A(B), \forall B \subseteq A$, $Bl(B)$ values from BBA $m(B), \forall B \subseteq A$.



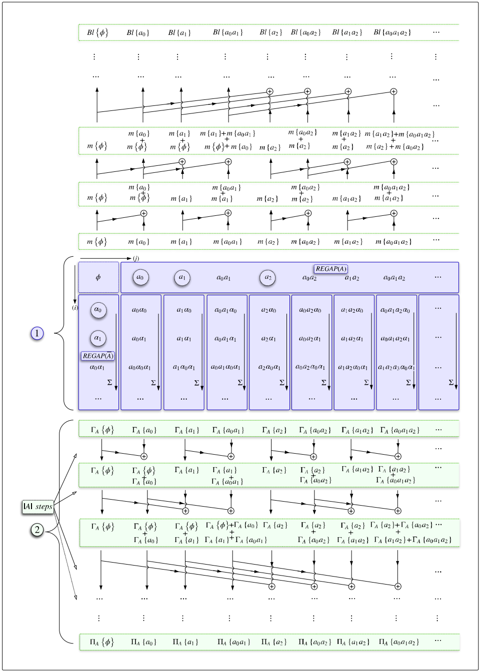
DS-Conditional-All Model

- Construct the DS-Matrix as before with $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$ and $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.
- Utilize the fast Möbius transformation (FMT)[18,19].
- Perform following computations $\boldsymbol{\Gamma_A(B)}$, $\forall \boldsymbol{B} \subseteq \boldsymbol{A}$, $\Pi_A(B)$ values from $\Gamma_A(B)$, $\forall B \subseteq A$, $Bl(B)$ values from BBA $m(B)$, $\forall B \subseteq A$.



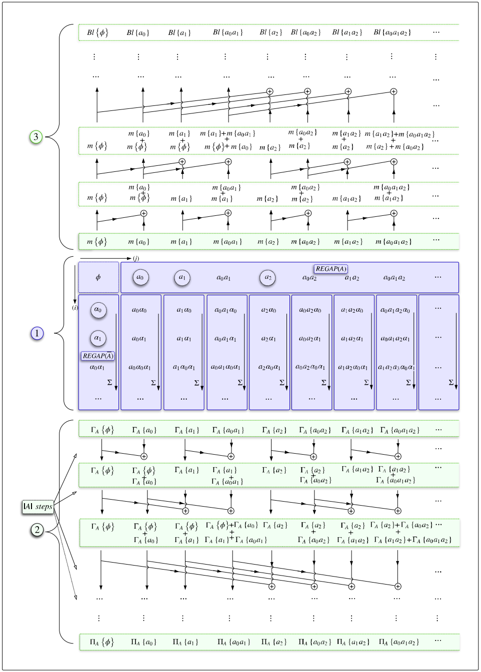
DS-Conditional-All Model

- Construct the DS-Matrix as before with $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$ and $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.
- Utilize the fast Möbius transformation (FMT)[18,19].
- Perform following computations $\Gamma_A(B), \forall B \subseteq A$, $\mathbf{\Pi_A(B)}$ **values from** $\mathbf{\Gamma_A(B), \forall B \subseteq A}$, $Bl(B)$ values from BBA $m(B), \forall B \subseteq A$.



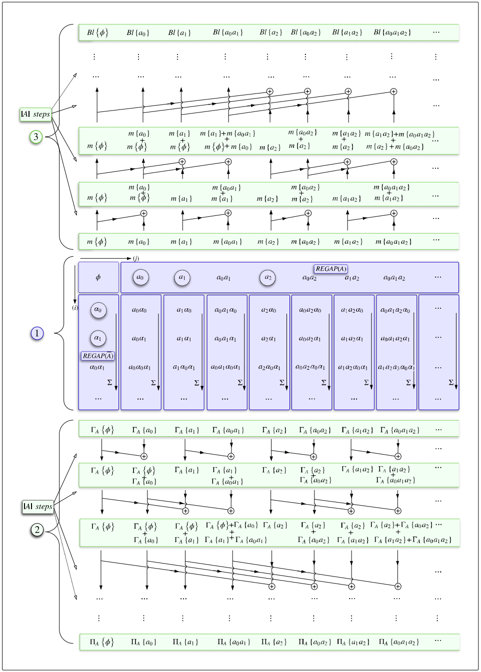
DS-Conditional-All Model

- Construct the DS-Matrix as before with $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$ and $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.
- Utilize the fast Möbius transformation (FMT)[18,19].
- Perform following computations $\Gamma_A(B), \forall B \subseteq A$, $\Pi_A(B)$ values from $\Gamma_A(B), \forall B \subseteq A$, $\boldsymbol{Bl(B)}$ **values from BBA** $\boldsymbol{m(B), \forall B \subseteq A}$.



DS-Conditional-All Model

- Construct the DS-Matrix as before with $A = \{a_0, a_1, \ldots, a_{|A|-1}\}$ and $\overline{A} = \{\alpha_0, \alpha_1, \ldots, \alpha_{|\overline{A}|-1}\}$.
- Utilize the fast Möbius transformation (FMT)[18,19].
- Perform following computations $\Gamma_A(B)$, $\forall B \subseteq A$, $\Pi_A(B)$ values from $\Gamma_A(B)$, $\forall B \subseteq A$, ***$Bl(B)$* values from BBA *$m(B)$*, $\forall \boldsymbol{B} \subseteq \boldsymbol{A}$.**

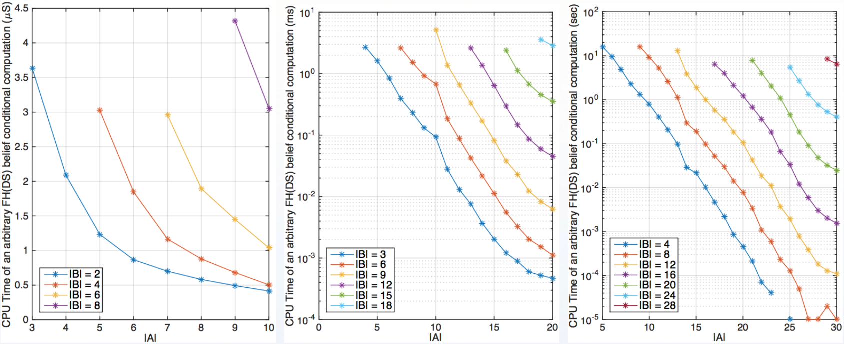


DS-Conditional-All Computational Model

- Look at Propositions 4 and 5.

Use	To Compute	Complexity	
		Time	Space
Dempster's and FH Conditional Beliefs of All Propositions:			
$REGAP(\overline{A})$	$Bl(\overline{A}) = \Gamma_A(\varnothing) = \Pi_A(\varnothing)$	$\mathcal{O}(2^{\lvert\overline{A}\rvert})$	$\mathcal{O}(2^{\lvert\Theta\rvert})$
(1) $REGAP(\overline{A}) \otimes B,\ \forall B \subseteq A$	$\Gamma_A(B),\ \forall B \subseteq A$	$\mathcal{O}(2^{\lvert\Theta\rvert})$	$\mathcal{O}(2^{\lvert\Theta\rvert})$
(2) $REGAP(A)$	$\Pi_A(B),\ \forall B \subseteq A$ from the FMT	$\mathcal{O}(2^{\lvert A\rvert} \times \lvert A\rvert)$	$\mathcal{O}(2^{\lvert A\rvert})$
(3) $REGAP(A)$	$Bl(B),\ \forall B \subseteq A$ from the FMT	$\mathcal{O}(2^{\lvert A\rvert} \times \lvert A\rvert)$	$\mathcal{O}(2^{\lvert A\rvert})$
Dempster's Conditional Mass of an Arbitrary Proposition:			
$REGAP(\overline{A})$	$Bl(\overline{A}) = \Gamma_A(\varnothing) = \Pi_A(\varnothing)$	$\mathcal{O}(2^{\lvert\overline{A}\rvert})$	$\mathcal{O}(2^{\lvert\Theta\rvert})$
(1) $REGAP(\overline{A}) \otimes B,\ \forall B \subseteq A$	$\Gamma_A(B),\ \forall B \subseteq A$	$\mathcal{O}(2^{\lvert\Theta\rvert})$	$\mathcal{O}(2^{\lvert\Theta\rvert})$

Experiments



(a) $|\Theta| = 10$ (b) $|\Theta| = 20$ (c) $|\Theta| = 30$

Figure: CPU time for arbitrary FH (Dempster's) belief conditional computation versus $|A|$ for different $|B|$ values (when $|\Theta| = 10$, $|\Theta| = 20$, and $|\Theta| = 30$).

- For a given FoD size, we selected a random set of focal elements, with randomly selected mass values, and conducted 10,000 conditional computations for randomly chosen propositions A and $B \subseteq A$.

Experiments

Method → Conditional →		DS-Conditional-One Model FH or Dempster's			Dempster's
FoD		$Bl(B\|A)$ or $Bl(B\\|A)$	$Bl(B\|A)$ or $Bl(B\\|A)$	$m(B\|A)$ or $m(B\\|A)$	$m(B\\|A)$
$\|\Theta\|$	Max. $\|\mathfrak{F}\|$	**(Arbitrary)**	**(All)**	**(All)**	**(Arbitrary)**
2	3	0.0008	0.0016	0.0024	0.0008
4	15	0.0008	0.0057	0.0068	0.0008
6	63	0.0009	0.0189	0.0208	0.0009
8	255	0.0011	0.0707	0.0758	0.0010
10	1,023	0.0016	0.3038	0.3208	0.0012
12	4,095	0.0033	1.5535	1.6206	0.0016
14	16,383	0.0095	<u>15.0000</u>	<u>17.1429</u>	0.0030
16	65,535	0.0323	<u>131.8750</u>	<u>136.8750</u>	0.0074
18	262,143	0.1223	<u>1,072.2200</u>	<u>1,077.7800</u>	0.0218
20	1,048,575	0.4724	<u>8,670.0000</u>	<u>8,698.0000</u>	0.0771
22	4,194,303	3.1889	<u>71,115.9000</u>	<u>73,942.3000</u>	0.2853
24	16,777,215	<u>18.7807</u>	<u>653,268.0000</u>	<u>660,883.0000</u>	0.6467
26	67,108,863	<u>83.0787</u>	<u>1.6334 cpu hours</u>	<u>1.6915 cpu hours</u>	1.1744
28	268,435,455	<u>338.2960</u>	***	***	<u>31.2735</u>
30	1,073,741,823	<u>1,509.5000</u>	<u>***</u>	<u>***</u>	<u>111.2910</u>

Table: DS-Conditional-One model. Average computational times with DS-COCA library (*ms*) (******* denotes computations not completed within a feasible time or space requirement. Conditional computations for larger FoDs were done a supercomputer (`https://ccs.miami.edu/pegasus`) (<u>underlined</u> in Tables)).

Experiments

Method → / Conditional →		DS-Conditional-All Model: FH or Dempster's	DS-Conditional-All Model: FH or Dempster's	DS-Conditional-All Model: Dempster's	Specialization Mat.: Dempster's
FoD: $\|\Theta\|$	FoD: **Max.** $\|\mathfrak{F}\|$	$Bl(B\|A)$ or $Bl(B\|\|A)$ **(All)**	$m(B\|A)$ or $m(B\|\|A)$ **(All)**	$m(B\|\|A)$ **(All)**	$m(B\|\|A)$ **(All)**
2	3	0.0011	0.0014	0.0015	0.0015
4	15	0.0014	0.0018	0.0020	0.0070
6	63	0.0026	0.0034	0.0031	0.0767
8	255	0.0067	0.0091	0.0054	1.1264
10	1,023	0.0211	0.0303	0.0135	98.4795
12	4,095	0.0770	0.1133	0.0427	1,581.8300
14	16,383	0.2950	0.4378	0.1532	24,847.0000
16	65,535	1.1592	1.7243	0.5814	396,860.0000
18	262,143	6.5901	9.2096	2.3430	1.7637 cpu hours
20	1,048,575	26.7221	39.0397	9.3537	***
22	4,194,303	112.4180	166.0070	43.5348	***
24	16,777,215	500.3420	689.8700	233.6080	***
26	67,108,863	2,239.2400	2,908.7000	1,118.9500	***
28	268,435,455	9,273.8100	12,406.4000	4,976.9700	***
30	1,073,741,823	42,087.2000	52,055.8000	25,354.9000	***

Table: DS-Conditional-All model versus specialization matrix based method. Average computational times (*ms*) (******* denotes computations not completed within a feasible time or space requirement. Conditional computations for larger FoDs were done a supercomputer (`https://ccs.miami.edu/pegasus`) (underlined in Tables)).

Time and Space Complexity Comparison

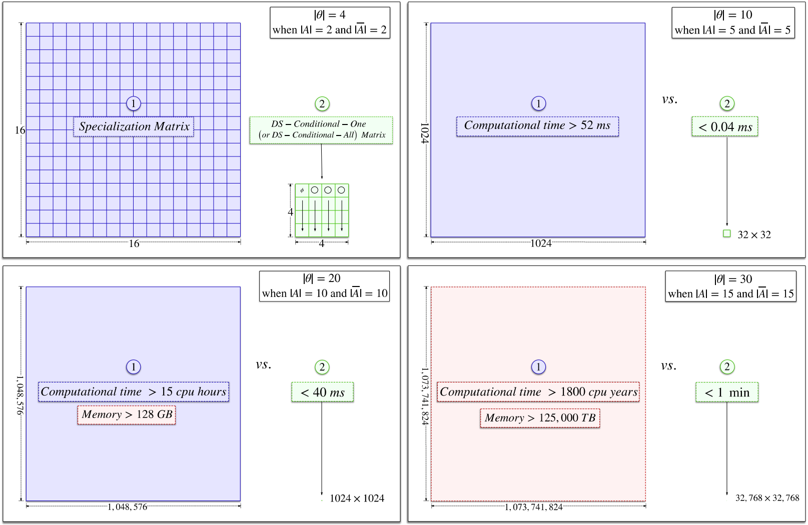


Figure: Time and space complexity comparison of DS-Conditional-One (or DS-Conditional-All) model with the specialization matrix approach (Theoretical computational times calculated assuming 10,000,000 iterations per second).

Important Remarks

- Reasons for significant performance in performance:
  - Smaller matrix size (corresponding to the BoE only).
  - No matrix multiplications (only additions are involved).
  - Repetitive computations avoided.
  - Access operation of a focal element takes only constant time.
- An outcome of this research: *DS-COCA* library[20].

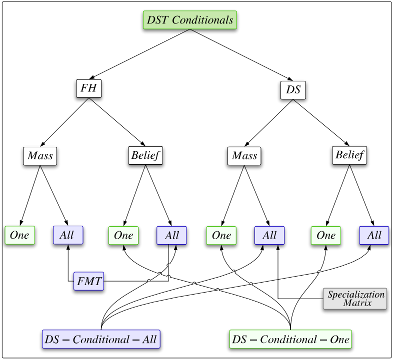


Figure: Best use of DST conditional computation models.

- These models can also be used for the following purposes:
  - Visualization and analysis of the conditional computation process.
  - Real-time evidence fusion and uncertainty reasoning applications[21].

Outline

Operations on Dynamic Frames: Removing Singleton

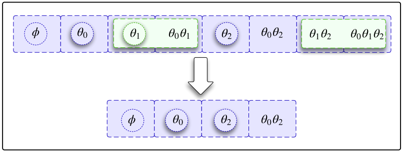


Figure: DS-Vector

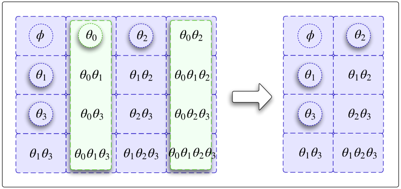


Figure: DS-Matrix

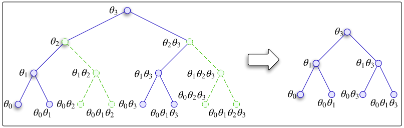


Figure: DS-Tree

Outline

DST Fusion Strategies

- An efficient algorithm for the Dempster's Combination Rule (DCR).
- Address DCR's limitations regarding conflicting evidence.
- Develop an effective strategy to apply the DCR to multiple BoEs.

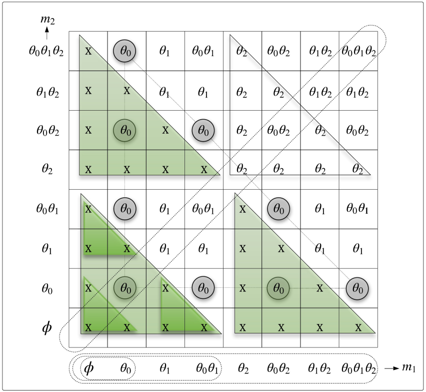


Figure: DCR for 2 BoEs.

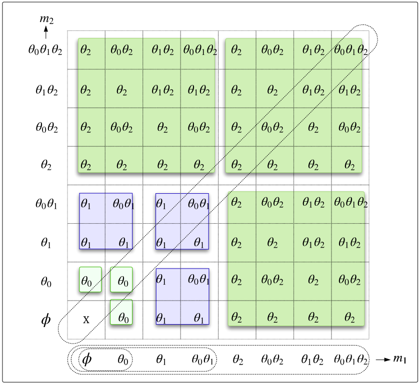


Figure: DST conjunctive combination of 2 BoEs.

DST Fusion Strategies

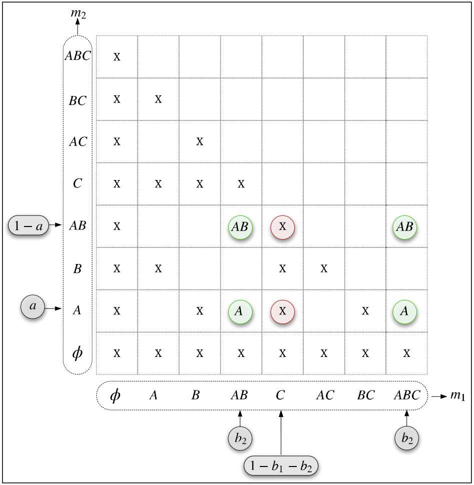


Figure: Visualization of conflicting evidence in DCR.

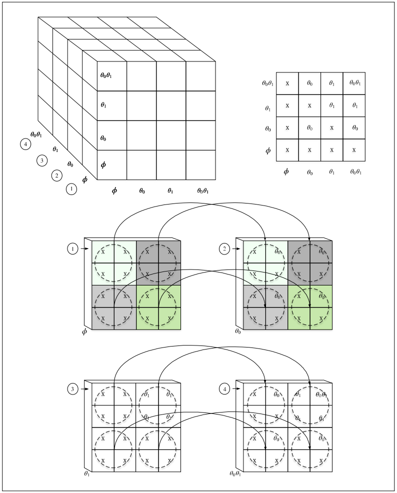


Figure: DCR for 3 BoEs.

DST Fusion Strategies

- We are also working on developing efficient algorithms for the following DST fusion strategies:
  - Conditional Update Equation (CUE)[22].
  - Conditional Fusion Equation (CFE)[23].
  - Pignistic transformation[24].
- We plan to develop efficient algorithms and data structures to work with special low density BoEs:
  - Dirichlet belief functions[25].
  - Consonant belief functions[8].
  - Single focal element (SFE) BoEs.
  - Other low density BoEs.
- We are conducting a wide range of experiments with dynamic operations to improve the computational performance.

Baseline Network Selection in InSAR

- Synthetic aperture radar interferometry (InSAR)[26] is an important technique that can measure terrain deformation with high precision.
- InSAR finds application include geophysical monitoring, including earthquakes, volcanic eruptions, landslides, and hydrological subsidence[27,28].
- We plan to use our newly developed algorithms and data structures to develop an efficient selection criterion to identify the best baseline network, which is a primary component in InSAR processing.
- We have already conducted two research studies:
  - Network selection using centrality concepts[29].
  - Identifying higher quality networks using deep learning[30] techniques.
- Both these initial steps have yielded promising results.

Baseline Network Selection in InSAR

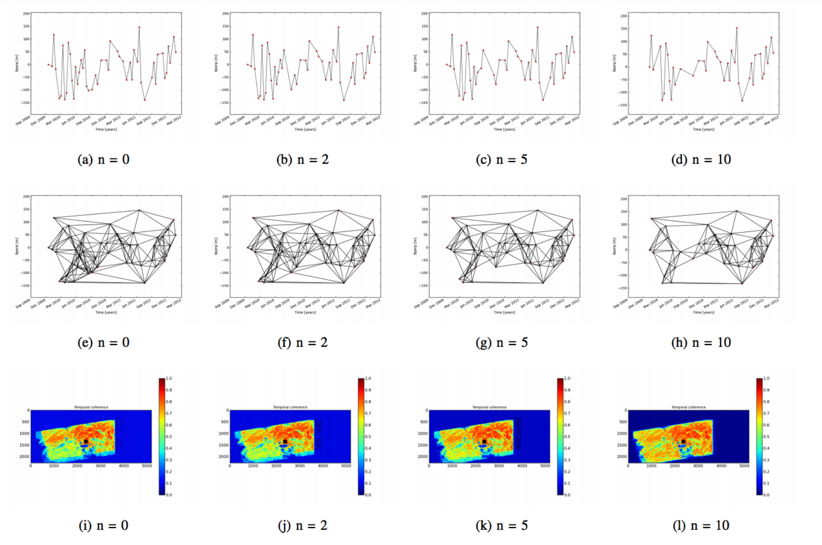


Figure: Network selection using flow-betweenness centrality. (a)-(d) Network of baseline history. (e)-(h) Network of interferograms. (i)-(l) Map of temporal coherence.

Baseline Network Selection in InSAR

- 1200 networks were used to train and test the AI prototype of network quality measurement.
- Each network contains 5 blue and 5 green nodes.
- The value of the edges between the same color nodes is +1 and otherwise -1.
- The trained AI prototype gave 99.3% accuracy.
- The ongoing challenging task is to adapt uncertainty reasoning capabilities for a robust strategy for practical InSAR processing.

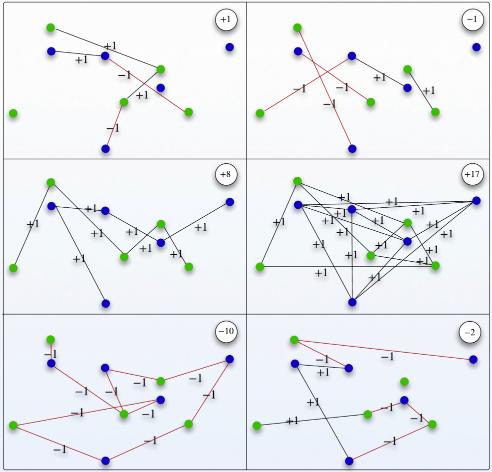


Figure: Six out of 1200 networks used in the deep learning strategy.

Effective DST Visualizations

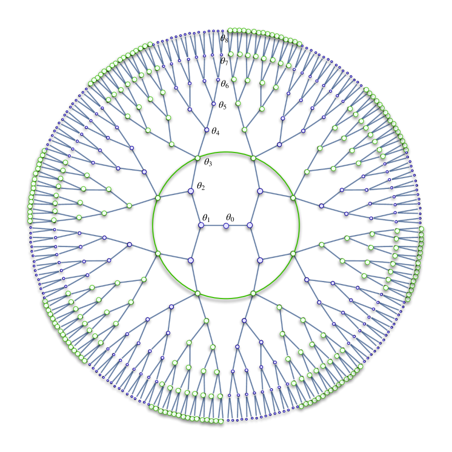


Figure: **DS-LASIC (DS-Layered Symmetric Clustering) Diagram:** Dynamic BoE representation as a layered symmetric clustering diagram when $|\Theta| = 9$.

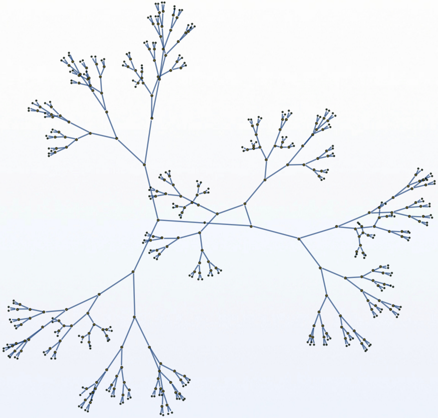


Figure: **DS-TRISEV (DS-Three Dimensional Spring Electrical Visualization) Model:** 3D dynamic BoE representation using spring electrical[31] model when $|\Theta| = 9$.

Computational Libraries

(a) BCL: Belief Computation Library[16]

(b) CCL: Conditional Computation Library[32]

(c) DS-COCA Library: DS-Conditional-One and DS-Conditional-All[20]

- These libraries are being improved and computational libraries for other findings are being developed.

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Thank You!
Questions?